Higgs potential and fundamental physics

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Physics associated with the Higgs potential is rich and very interesting. I believe it deserves efforts aimed at explaining it to high school students even though the subject may appear complex, even formidable at times. Before we embark on this task we might benefit from an attempt to summarize some of the basics in this field so we could better see the beauty and the challenges that go along with it. In this short note I will limit myself to several topics, discussing electroweak phase transition, QCD phase transition (chiral symmetry breaking), naturalness problem and vacuum stability. I do not deal directly with the most obvious topic of the Higgs mechanism of electroweak symmetry breaking.

1 Introduction

The Higgs potential $V(H)$ for a simple case of a real scalar field H can be written as

$$V(H) = \lambda(H^2 - v^2)^2 = \lambda H^4 - 2\lambda v^2 H^2 + \lambda v^4 \tag{1}$$

where H is the Higgs field. Taking $\frac{\partial V(H)}{\partial H} = 0$ we find the minimum of the potential at $H = \pm v$. This defines the current vacuum in the Universe. When we expand the field H about the minimum as $H = h + v$ (the standard procedure), we get

$$V(h) = \lambda[(h + v)^2 - v^2]^2 = \lambda h^4 + 4\lambda v h^3 + 4\lambda v^2 h^2 \tag{2}$$

The last term has the form of a mass term, $M_H^2 h^2/2$, hence the Higgs boson mass is

$$M_H^2 = 8\lambda v^2 \tag{3}$$

Both v and λ paramaters are determined experimentally through the measurement of the Fermi constant G_F and the Higgs boson mass $M_H = 126$ GeV, yielding

$$v \doteq 246 \text{ GeV}, \quad \lambda \doteq 0.13 \tag{4}$$

The potential is shown in Fig.1 for two values of the Higgs mass: the measured value $M_H = 126$ GeV and $M_H = 800$ GeV. We note that the Higgs mass squared
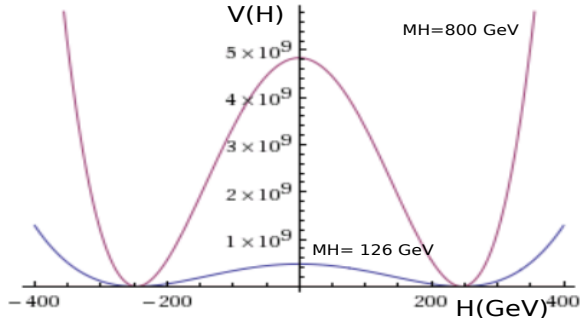
Figure 1: Higgs potential in GeV^4 as a function of the Higgs field average value H in GeV for two values of the Higgs mass: $M_H = 126$ GeV (blue line) and $M_H = 800$ GeV (purple line).

is the curvature of the Higgs potential at the minimum $H = v$ as one can see from $\frac{\partial^2 V(H)}{\partial H^2}|_{H=v} = 8\lambda v^2 \equiv M_H^2$. The valley around the minimum is thus flat for low M_H and narrow with steep sides for large M_H, see Fig.1. This point will have significance for the naturalness problem to be discussed below.

$V(H)$ can be interpreted as the Higgs vacuum energy density (energy density of the empty space). For our choice of the potential, the vacuum energy density is zero at the minimum $H = v$. However, for the potential energy it is the difference that matters, not the absolute value and thus the relevant contribution is the constant term in Eq. 1 (the size of the hill at $H = 0$), $\lambda v^4 = 4.8 \times 10^8$ GeV^4. From cosmology we have a vacuum energy density that is roughly 55 orders smaller [3] and this huge difference is a mystery, the cosmological constant problem.

2   Electroweak phase transition

Elementary particles became massive in the early Universe when the Higgs potential took the form shown in Fig.1 as a result of the electroweak phase transition which occured at $\sim 10^{-11}$ s after the Big Bang. To reconstruct effective potential at high temperatures present at that time, either perturbative calculations or lattice simulations are performed within finite temperature effective field theory. The first order temperature correction to the Standard model potential of Eq.1 is proportional to T^2, leading to the effective potential [1, 2]

$$V(T, H) \;\;=\;\; \lambda(H^2 - v^2)^2 + b\,T^2 H^2 \tag{5}$$

where b is the coefficient which depends on the couplings of the Standard model particles to the Higgs field. This potential is shown in Fig.2 left. Here $T_c = \sqrt{2\lambda v^2/b}$ is the critical temperature of the phase transition. For $T > T_c$ the potential is symmetric with the minimum at $H = 0$. At $T = T_c$ the valley
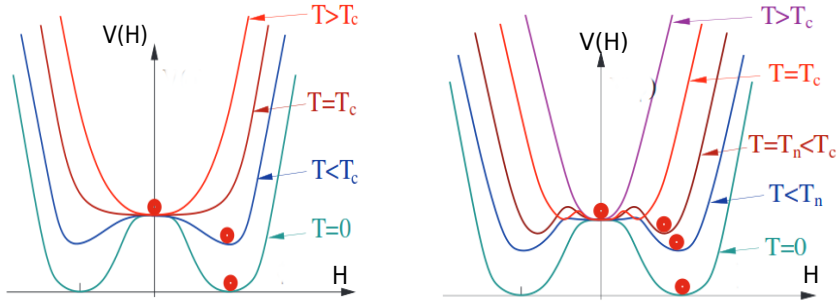
Figure 2: Left: Crossover or second order phase transition [1]. Right: First order phase transition [1].

becomes very flat and as soon as $T < T_c$, the potential developes minima at $H > 0$ and $H < 0$. Finally, for $T = 0$ the minima (which move away from $H = 0$ during cooling) arrive at $H = \pm v$ and the potential becomes identical with the one in Fig.1. This kind of phase transition is called second order or crossover.

Another possibility is the first order phase transition depicted in Fig.2 right. Here at $T = T_c$ we have 3 degenerate minima: the original one at $H = v$ and the two minima at nonzero H seperated by a barrier from the central minimum. At $T = T_n$ the universe tunnels through the barrier and takes its position at one of the nonzero H minima. The first order scenario could be realized through the second order temperature correction which modifies the effective potential as

$$V(T, H) \quad = \quad \lambda(H^2 - v^2)^2 + b\,T^2H^2 + a\,TH^3 \qquad (6)$$

where a is a constant.

The nature of the phase transition is very interesting for cosmology. During second order (or crossover) transitions Universe is constantly at thermal equilibrium and the system thus loses the memory of the initial state from which it began. Thus, we do not expect remnants at lower temperatures $(T < T_c)$ from the unbroken phase [1]. On the other hand for the first order transition, we could get remnants possibly observable in astrophysical data. Also the hypothesis that matter-antimatter asymmetry is explained by the electroweak phase transition requires the first order transition.

To determine the nature of the phase transition within the Standard model, one has to go beyond the perturbative approximations of Eqs. 5,6. As more precise lattice studies show, the crucial parameter is the Higgs mass. For $M_H = 126$ GeV the phase transition is crossover for the Standard model. This, however, might change if we go beyond the Standard model. New physics could still induce the first order phase transition.

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We conclude with a note that the numerical value of the critical temperature for the Standard model in the approximations of Eqs. 5,6 is around $T_c \approx 160$ GeV.

3 QCD phase transition

At $t \sim 10^{-6}$ s after the Big Bang the Universe underwent the QCD phase transition during which free quarks and gluons (quark-gluon plasma) became confined within hadrons. At almost the same time the chiral symmetry was broken and nucleons and other hadrons gained their masses. The nucleons consist of three quarks with current-quark masses of $m_u \sim 3 - 5$ MeV and $m_d \sim 7 - 9$ MeV for the up quark and the down quark, respectively. These masses are attributed to the Higgs field (electroweak phase transition). Since nucleons have masses of $938 - 939$ MeV, the Higgs field is responsible for only 2% of that, whereas 98% derives from the chiral symmetry breaking.

Perhaps surprisingly, the relevant physics can be described formally by the same scalar potential as the Higgs potential of Eq. 1,

$$V(H) \quad = \quad \lambda(H^2 - v^2)^2, \tag{7}$$

except that parameters v, λ and the field H have to reinterpreted[1]. The QCD vacuum is defined by the pion decay constant $v = 90$ MeV, H is the scalar field which is, however, not elementary like the Higgs field but is composed of $u\bar{u}$ and $d\bar{d}$ pairs (condensate). The quantum of this field, the so-called σ meson is the QCD sibling of the Higgs boson. Like the Higgs boson, σ has been searched for for decades until it was finally concluded around 2010-2013 that it had been observed at MAMI (Mainz) with the mass of about 660 MeV [4]. The corresponding value of λ is 6.7. Obviously, the potential also looks like a Mexican hat (Fig. 3) except that the scale is different with units now in MeV rather than GeV.

The nature of the QCD phase transition is also under an intensive scrutiny in the heavy ion community. Both scenarios depicted in Fig. 2 are possible, depending on the energy of colliding heavy ions. For LHC we probe the region in the phase diagram which corresponds to the crossover phase transition and this we believe was true also for the early Universe.

While we did not explain how nucleons get their masses (we have not done that for electroweak symmetry either), we drew a parallel between the two phase transitions. If we decided to go further in this analogy, we might argue that even the Higgs field may not be elementary but rather a condensate composed of, e.g., techniquarks. This is the idea behind technicolor and related theories of dynamical symmetry breaking, which still represent a serious alternative to supersymmetry.

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[1]This is the picture we get in the linear σ model which is an effective model of QCD at few hundred MeV.
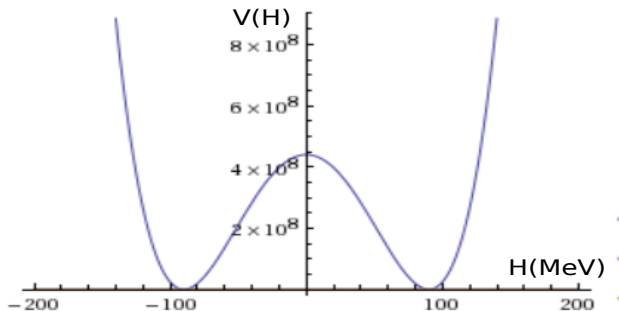
Figure 3: QCD scalar potential in MeV^4 as a function of the σ field average value H in MeV. The difference from the Higgs potential is in the scale (MeV vs GeV) and the fact that the scalar field here is the condensate of quark-antiquark pairs.

4 Naturalness problem

Difficulties known as naturalness problem (also hierarchy problem) arise when we try to calculate the Higgs potential in the Standard model. I will try to summarize here an excellent explanation by Matt Strassler [5] (in case I fail, go to his original treatment). The calculation involves sum of many contributions, such as energy from the quantum fluctuations of the Higgs field itself, energy from the fluctuations of the top quark field, W field, Z field and so on for all the fields which interact with the Higgs field. The individual contributions are shown schematically in Fig.4 as a function of the Higgs field's value from $H = 0$ up to some large value $H = v_{max}$ which is the boundary between where the Standard model is applicable and where it is not.

The first row of Fig.4 shows contributions from the Standard model fields. Each of these can be calculated unlike the contributions from new physics beyond the Standard model shown in the second row which we do not know how to calculate. Both contributions in the first row and the second row are big, in fact much bigger than $V(H)$ in Fig.1 *if v_{max} is much larger than v*. Not only they are big but they also vary a lot with H. Moreover, the contribution of each field appears unrelated to the other contributions (true for the first row and natural assumption for the second row). The problem is that when we add up all the contributions, each of them big and varying a lot, we must somehow get what experiment tells us: that $v = 246$ GeV and $M_H = 126$ GeV, that is we have to end up with the green curve in the third row[2]. The green curve is incredibly flat compared to the individual contributions. As Matt Strassler puts it, it is as though you piled a few mountains from Montana into a deep valley

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[2]The green curve is (almost) the same thing shown in Fig.1 except that the Higgs potential was shifted by a constant value to negative values which has no physical importance for the subject discussed here.

in California and ended up with a plain as flat as Kansas. Recall that the flat valley around the vacuum $H = v$ corresponds to low M_H while large M_H leads to steep sides of the valley. In other words, the individual contributions which vary a lot ('steep sides') would most likely induce very large M_H, close to v_{max}.

Moreover, each of individual contributions in Fig. 4 has minima and maxima at Higgs field values that are either at zero or somewhere around v_{max}, and adding those curves together, you will find that the sum of those curves is a curve that also has its minima and maxima at a substantial fraction of v_{max} or at zero, but not at $v = 246$ GeV (very very close to zero)[5]. So it is both the Higgs mass and the minimum v which seem unnaturally small and fine-tuning must be invoked to ensure cancellations between 'Montana peaks' and 'California valleys'.

We stress that all this is true only to the degree that v_{max} is much larger than v. For v_{max} close to v the fine-tuning is not significant. The naturalness problem has been the driving force in particle physics for decades. One class of solutions (such as supersymmetry) concentrates on the symmetries which ensure that individual contributions in Fig.4 are related in pairs - two curves have almost equal size and opposite signs leading to the sum which is almost flat (at least near our vacuum). Another class of solutions (such as technicolor above) argues that v_{max} is in fact close to v.
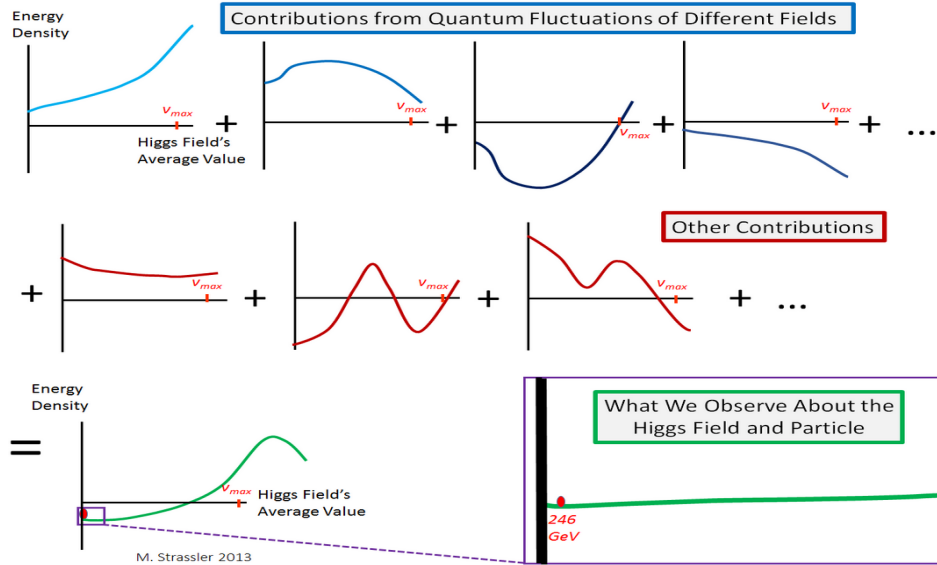

Figure 4: Schematic contributions to the Higgs potential from the Standard model (first row) and from new physics (second row). If $v_{max} \gg v$, this requires a very precise cancellation between the known and unknown sources of energy, one that is highly atypical of quantum theories [5].

5 Is our vacuum stable?

As we have seen in the previous section, when contributions to the Higgs potential from the quantum fluctuations of the fields of the Standard model (SM) particles are summed up, we get the green curve in Fig. 4. Note that the green curve changes its behaviour from increasing to decreasing for very large values of the average Higgs field H. The crucial question is the continuation of the green curve for even larger H. Will it follow the curve indicated in Fig. 5 (Left) as 'Stable'? Or will it be more like the curve marked 'Metastable'? The short answer is that *assuming no new physics until Planck scale* $\sim 10^{18}$ GeV, the SM calculations indicate that there is a (very large) value of the Higgs field (H_c) beyond which the Higgs potential becomes negative and the second (true minimum) develops. This makes our current vacuum metastable - Universe is ready to tunnel to the true minimum (with catastrophic consequences), albeit with very low probability.
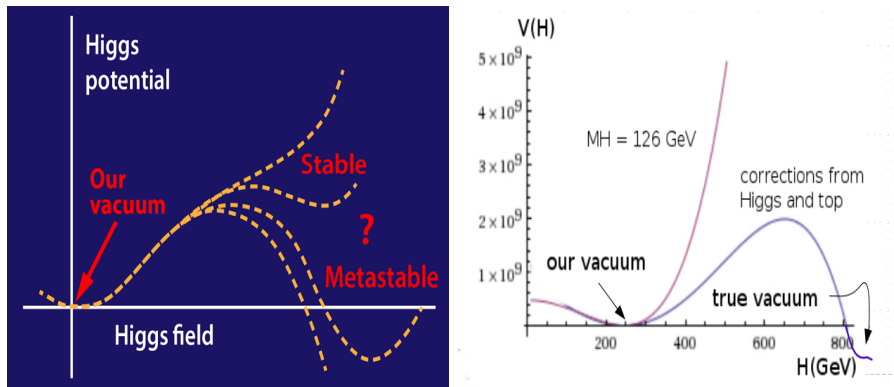

Figure 5: **Left:** If the Universe lies in the global minimum of the potential, then it is stable. But if the minimum is local and a deeper minimum exists, the vacuum is false, and the Universe might catastrophically tunnel out into the true vacuum state. Credit: APS/Alan Stonebraker. **Right:** SM tree-level Higgs potential of Eq. 1 (purple line) and SM Higgs potential corrected by the quantum corrections (blue line). Note that we used Eqs. 8 and 9 below with the top quark mass artificially set to $m_t = 210$ GeV for illustration purposes.

To show that this could be the case, note that the quantum corrections can be absorbed in the λ parameter of the SM Higgs potential of Eq. 1:

$$V(H) = \lambda(H)(H^2 - v^2)^2, \tag{8}$$

where $\lambda(H)$ became effectively a running constant, given in the one-loop approximation by [6]

$$\lambda(H) = \lambda(M_H) + \frac{1}{4\pi^2}\Big(-6\frac{m_t^4}{v^4} + 24\,\lambda(M_H)^2 + ...\Big)\ln(\frac{H}{M_H}) \tag{9}$$

with $\lambda(M_H) = 0.13$. Only the largest quantum corrections due to the top quark mass m_t and the Higgs field fluctuations (term $\lambda(M_H)^2$)) were included. The top quark with its large negative contribution induces the crucial change in the running of $\lambda(H)$ which becomes negative (and so does $V(H)$) at some large H_c.

We plot this effective Higgs potential (tree-level + two largest quantum fluctuations) in Fig. 5 (Right). To illustrate the effect qualitatively within the linear scale, we set the top quark mass to $m_t = 210$ GeV which yields $H_c \sim 800$ GeV. Note, however, that the actual value of H_c, which is very sensitive to experimentally measured values of M_H, m_t and also α_s and their uncertainties, is much larger. For current values of these parameters ($M_H = 125.1$ GeV, $m_t = 173.3$ GeV) the most sophisticated calculations [7] give $H_c \sim 10^{11}$ GeV, signalling that the Higgs potential develops a new global minimum at large Higgs field values, $H \sim 10^{17}$ GeV [8]. The height of the barrier between the two minima is as high as $\sim 10^{39}$ GeV^4.

To indicate an extreme sensitivity of the border between stability, metastability and instability[3] to m_t and M_H, we show in Fig.6 the results of the latest and most precise calculations of the position of our Standard model vacuum in the $m_t - M_H$ plane [7]. Our vacuum sits in the region of metastability but within just 2σ from the stability region.
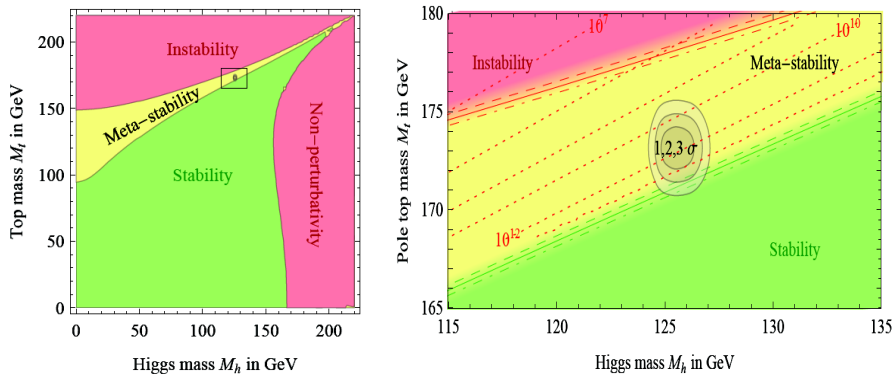

Figure 6: Regions of absolute stability, meta-stability and instability of the SM vacuum in the $m_t - M_H$ plane

For a calculation of the tunnelling probability of our Universe to the true vacuum, see for example, Ref. [9].

The decay of our metastable vacuum could be, at least in principle, catalysed by the cosmic ray collisions which could lead to an increased tunnelling probability. This question has been studied by Ref. [10]. Their results also indicate that "vacuum decay is very unlikely to be catalysed by particle collisions in accelerators; the total luminosities involved are simply far too low".

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[3]Instability means significant probability to tunnel to the true vacuum within the age of the Universe.

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